Learning opportunities for statistical literacy in German middle school mathematics textbooks

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The development of statistical literacy is an important goal for middle schools, where statistics education mostly takes place within mathematics classrooms. Here, textbooks provide the most important tool for many teachers, guiding the content of their lessons. However, little is known about the statistical content of middle school mathematics textbooks. This study reports on a qualitative document analysis of three German Grade 6 textbooks. The results show that a large majority of tasks in textbooks revolve around technical constructions of diagrams and calculations of measures. Less space is allocated towards more conceptually demanding tasks like interpreting models or analysing and reflecting statistical arguments. This implies that teachers need to actively adapt their textbooks in order to unlock the potential for developing statistical literacy of these textbooks.

Keywords: Statistics education research, statistical literacy, textbooks, document analysis, middle school.

Introduction

In recent years, researchers in statistics education research have elaborated the importance of statistical literacy for aspects of digitalization such as big data and machine learning (François et al., 2020). It is becoming increasingly clear that statistical literacy cannot be reduced to a skill of specialists, but rather will be important for every citizen in the 21st century (Wild, 2017). Therefore, the development of statistical literacy becomes an important task for middle schools.

This is a challenging task, as in many countries, statistics is considered only a small part of middle school mathematics instruction, and only very limited time can be allocated to statistics (Zieffler et al., 2018). Although researchers in statistics education research have begun to address this issue, insights into how statistical literacy can be developed in middle schools are limited yet (Büscher, in press). Thus, teachers have to contend themselves with the learning opportunities for developing statistical literacy that are provided by their mathematics textbooks.

This makes mathematics textbooks an important object of study. The content of textbooks largely defines the content of mathematics classrooms, as content that is not included in textbooks generally is not taught in class (Stein et al., 2007). Textbooks need to provide teachers with the suitable didactical instruments for developing statistical literacy. This study aims to provide insights into the learning opportunities for statistical literacy afforded by middle school mathematics textbooks.

Theoretical background

Statistical literacy as selective and imaginative readings of statistical information

Statistical literacy generally refers to the ability to understand and to critically evaluate statistical information presented in everyday media like newspapers, articles, or infographics (Gal, 2002). Whereas earlier conceptualizations of statistical literacy mostly related citizens to the role of "data

consumers" (Gal, 2002), researchers recently have emphasized that statistical literacy also requires the development of skills more in line with data producers (Weiland, 2017). In order to integrate the two perspectives of data producer and data consumer, this study conceptualizes statistical literacy as the two processes of *selective and imaginative reading of statistical information* (Figure 1, Büscher, in press). *Selective reading* refers to the process producing concise statistical arguments. During this process, selective activities reduce the available information (illustrated by the progressively smaller boxes in Figure 1): A phenomenon is *encoded* into data by selecting only certain aspects that are then quantified. The data are then *abstracted* into a model by mathematizing certain relationships within the data. Finally, the model is *interpreted* by combining some of these relationships with a claim about the phenomenon under investigation, resulting in a statistical argument.

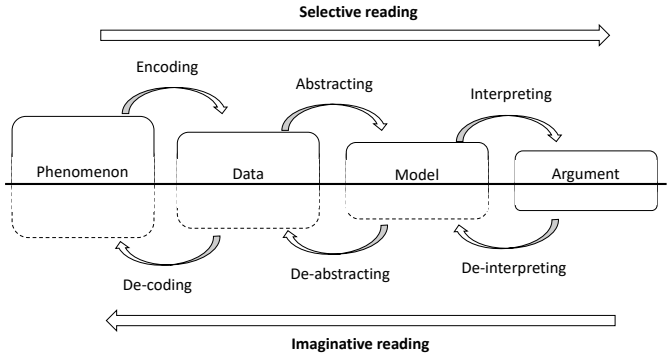

Figure 1: Statistical literacy encompasses activities of selective and imaginative reading

Crucially, a reader that is presented with a statistical argument in, for example, a social media post, likely does not have access to the underlying model or data. In order to critically evaluate the statistical argument, one has to revert the acts of selective reading through *imaginative reading* of what could have resulted in the argument (the dashed boxes in Figure 1). A statistical argument has to be *de-interpreted* to intuit the underlying model behind the argument, for example by guessing the type of measure of centre that an argument simply refers to as "average". Such a model only represents relationships in data, not the data themselves. The reader has to *de-abstract* from the model to imagine possible data behind the model, and what features of these data a median might or might not represent well. And finally, one has to recognize that the data only provide a quantified description of some aspects of the phenomenon that were obtained through certain methods of data collection. A *decoding* of the data might reveal important aspects that cannot be captured by the data. In this way, imaginative reading aims to discover possible causes and possible limitations of a statistical argument even if crucial information is missing.

This specification of the learning content of statistical literacy allows to decompose the larger construct into smaller activities that each can be the object of focused instruction. Instead of a holistic approach, teachers can create focused learning opportunities for each of the activities of selective and imaginative reading. This should not be taken as the claim that these activities should always be treated separately. Still, by identifying smaller activities, this conceptualization allows to identify the potential contributions to statistical literacy in many statistical tasks which are presented in textbooks.

Textbooks in statistics education research

Textbooks have a large impact on the enacted curriculum of schools, and Weiland (2019) proposes that this is especially true for statistics, where teachers have little prior experience. Thus, they might tend to adhere to the textbook more closely with statistics than with other subjects. In his study on United States high school textbooks, Weiland (2019) investigates what kinds of contexts are supplied in textbooks and how they are used. He finds that the contexts used "generally go no further than those typical of 'small talk', such as the weather, sports, personal preferences, or related to work or business" (Weiland, 2019, p. 32). He instead calls for textbooks to feature controversial socio-political issues to prepare students to be critical citizens. Tran and Tarr (2018) also investigate US high school textbooks, focusing on the complexity of the investigation of bivariate data in textbook tasks. They find that students are not required to formulate their own statistical questions, but are always given a fixed question in the tasks. Most of the time, students are provided with the data, which generally consists of fewer than 20 values and show no "messy" features like missing values. Thus, the textbooks provide little learning opportunities for organizing real, unstructured data.

Apart from these studies, not much research could be found that investigate the statistics content of textbooks. Under the statistical literacy perspective employed in this study, the existing studies suggest that the statistical arguments about contexts in the textbooks are uncontroversial, and thus might not motivate a deeper investigation of the sources of possible controversies through imaginative reading. Where selective readings are elicited, they are performed in a very fixed way, possibly emphasizing activities of abstracting over the more open activities of encoding and interpreting.

Research questions

A statistically literate citizen needs to be able to engage in activities of selective and imaginative reading. Textbooks need to provide teachers with suitable instruments to create learning opportunities for these activities. The little empirical knowledge available about textbooks suggests that textbooks might not be well equipped for this task, but further insights are needed. This study aims to provide a contribution by investigating the following research question:

(RQ 1) Which learning opportunities for activities of selective and imaginative reading are provided by German middle school textbooks?

(RQ 2) Which differences in learning opportunities exist between German middle school textbooks?

Method

Selection of textbooks

This study took the form of qualitative document analysis (Bowen, 2009). As a first step, relevant textbook series to be used in the analysis had to be selected. This proved a difficult task: In Germany, educational policy is a matter of the 16 federal states, which leads to variations in the mathematics curriculum and to state-specific textbooks. Additionally, textbook publishers generally do not disclose the market shares of their textbooks, so that little objective criteria exist for selecting textbook series for study. In the end, a theoretical sampling resulted in the selection of three textbook

series. Two of these series, *Lambacher Schweizer* ("LS", Jörgens, 2009) and *Elemente der Mathematik* ("EdM", Griesel et al., 2014), are textbook series used in German middle schools tracked for academic education. According to the publishers' description of their teaching conception, both textbook series provide a clearly structured learning progression with possibilities for differentiation and an emphasis on exercises. These series were selected to allow the identification of possible differences in learning opportunities for similar teaching conceptions. In contrast, *mathe live* ("ml", Glöckel et al., 2014) is a textbook series for integrated middle schools that introduce academic tracking only in later school years. According to the publisher, the teaching conception focuses on exploring mathematics in real-life situations and on individual approaches to mathematics.

From each textbook series, only the textbook for Grade 6 was included in the analysis. This decision was made because the mathematics curriculum for Grade 6 includes a relatively large part of statistics in relation to other grades. Content includes the construction and critical evaluation of various diagrams as well as measures of centre, which are important models for a statistically literate citizen. Only the chapters focusing on statistics were included in the analysis, and chapters focusing on probability were not included. This resulted in a data corpus of 371 tasks.

Data analysis

For data analysis, codes were assigned to each task according to the activities of selective and imaginative reading elicited by the tasks. For this, a coding scheme had to be developed in a multi-step approach consisting of deductive and inductive analytic phases. The assigned codes were compared and contrasted to identify possible incongruences in assigning the codes and to find the central categorial cuts between the codes. In the end, a coding scheme emerged that identified codes based on the *source* and the *goal* types of statistical information (phenomenon, data, model, argument). The source refers to the type of statistical information given in the task; the goal refers to the type of statistical information required as a solution to the task. The identification of the type of statistical information considered the language employed for giving the information: (a) statistical information on a *phenomenon* is characterized by rich descriptions of contextual knowledge without exact quantification. (b) Statistical information on *data* is characterized by atomic quantifications of certain aspects of the phenomenon. This includes categorical data as well as frequency data. (c) Statistical information on *models* refer to relationships within the data that are not reported by the data itself, but by additional models. Such models can be measures of centre as well as diagrams like pie charts, which can illustrate the proportional relationships between frequency data. Finally, (d) statistical information on *arguments* comprises justifiable claims about the phenomenon that are based on a model. Mere verbal descriptions of models are not considered statistical arguments; instead, an interpretative step has to be performed that situates the model in the larger phenomenon by incorporating additional context knowledge or by generalizing from the model.

Table 1 gives illustrates the final coding scheme. This scheme was applied in a final deductive step of analysis by identifying source and goal of the statistical information and assigning codes according to the coding manual in Table 1. Throughout the whole process, the assigned codes were discussed in the research team of the author and two colleagues. Not every task fit neatly into the coding scheme. These cases were discussed with the research team to provide a consensual validation of the coding.

In some cases, the team concluded that it was not possible to assign a code to a task. Such tasks often consisted of purely mathematical questions or were too unspecific to be clearly assigned to any activity. Although a skilled teacher might still use these tasks to develop statistical literacy, they could not be included in the analysis. Due to the complexity of the coding and the need for inductive development of the coding scheme with a research team, a test of inter-rater reliability was not applicable. Thus, the results of the analysis should be interpreted as exploratory findings on statistical literacy content in textbooks.

Table 1: The coding scheme for identifying activities of selective and imaginative reading

Activity	**Source**	**Goal**	**Example task**
Encoding	Phenomenon	Data	*- Conduct a survey about leisure activities in your class*
Abstracting	Data	Model	*- Given is a table with frequency data on leisure activities in class, Draw a pie chart.* *- Given is a table with daily sleep time in the class. Find the average.*
Interpreting	Model	Argument	*- Given is a chart on students' long jumps. Who should win the competition?*
De-Interpreting	Argument	Model	*- Given is a newspaper article. What does the author mean by "average"?* *- Tobias claims that boys are keener on sports than girls, Aylin disagrees. Find a justification for the claims in the data.*
De-Abstracting	Model	Data	*- The median daily sleep time is 8h. Which data could have produced the median?*
De-Coding	Data	Phenomenon	*- Given is data on youths' internet activities. Does this fit your own experiences?* *- Given is data on students' commutes to school by bus. Find a possible reason why some values are very high.*

Results

Table 2 provides an overview about the codes given in the analysis. The data is also visualized in Figure 2. Percentages do not add up to 100%, as some tasks were assigned multiple codes, and few tasks did not fit the coding scheme and were not assigned any codes.

Table 2: Codes assigned to the tasks in three textbooks

Textbook	**Tasks**	**Encoding**	**Abstracting**	**Interpreting**	**De-Interpreting**	**De-Abstracting**	**De-Coding**	**n/A**
EdM	130	14 *(11%)*	77 (59%)	9 (7%)	14 (11%)	13 (10%)	5 (4%)	19 (14%)
LS	146	8 (5%)	88 (60%)	3 (2%)	6 (4%)	35 (24%)	2 (1%)	15 (10%)
ml	95	3 (3%)	57 (60%)	11 (12%)	7 (7%)	11 (12%)	4 (4%)	6 (6%)

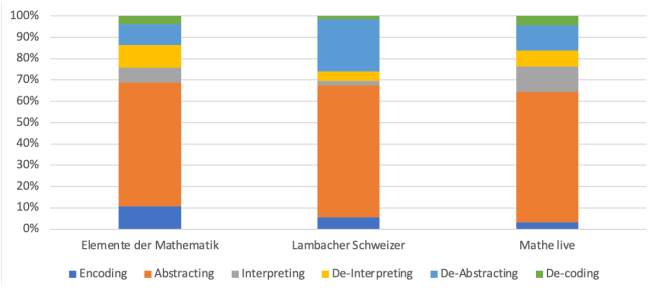

Figure 2: Textbooks heavily emphasize abstracting, but other activities vary

The data show that across all textbooks, the large majority of tasks (about 60%) elicit the selective activity of abstracting. The imaginative activity of de-coding shows the least occurrence in all of the textbooks, with only 1% to 4% of tasks eliciting this activity. Apart from these similarities, the other activities are featured very differently in the textbooks. *EdM* has the largest occurrence of encoding (11%) and de-interpreting (11%), *LS* by far has the largest amount de-abstracting (24%), and *ml* places a stronger emphasis on interpreting (12%) compared to the other textbooks.

Dimensions of differences between the textbooks

A comparison between the textbooks shows the very different learning opportunities for statistical literacy provided by the textbooks. A first dimension of the differences concerns the *technical nature of the tasks*. The activities of abstracting and de-abstracting in the textbooks are often elicited in tasks where a diagram has to be drawn based on given data (abstracting), or where possible data values have to be reconstructed given a measure of centre (de-abstracting). Both activities require no contextual knowledge, and can be solved in a very technical way. In *LS,* these two activities make up for 60% and 24% of the tasks. When combining the categories for both codes, 121 (83%) tasks remain that elicit abstracting or de-abstracting. This technical nature of the tasks could be the result of the exercise-driven approach of the textbook – although *EdM* also emphasizes exercises, and seems to provide more learning opportunities for more conceptually demanding tasks.

Another dimension of the differences could be provided by the different treatment of the activities of interpreting and de-interpreting. *EdM* and *ml* show a very different emphasis on these activities, with *EdM* favoring de-interpreting (11%) over interpreting (7%), and *ml* favouring interpreting (12%) over de-interpreting (7%). This difference could also be explained by the different approaches of the textbooks. *ml* claims to follow an exploratory approach with a focus on real-life contexts. This resonates with the activity of interpreting, in which conclusions about a phenomenon have to be drawn actively based on a model. In contrast, the activity of de-interpreting consists of searching for a fitting model for a given conclusion. As such, the differences between these two textbooks could be considered on a dimension of *exploratory vs. confirmatory data analysis*.

Summary

For all textbooks, the activity of abstracting is by far the activity with the most learning opportunities, as more than half of all tasks elicit this activity. Generally, all textbooks provide more learning opportunities for selective reading than for imaginative reading. This does not mean that the textbooks adequately support the development of selective reading, and do not support the development of imaginative reading. The overemphasis on abstracting also leads to only few learning opportunities

for the other activities of selective reading. The results suggest that the textbooks themselves are better suited for developing the single activity of abstracting than for whole processes of selective or imaginative reading. Generally, the activity of de-coding is almost absent from textbooks. However, learning opportunities for other selective or imaginative activities do exist in the textbooks. Where the textbooks do not mostly consist of technical tasks, they provide different learning opportunities for (de-)interpreting depending on an exploratory of confirmatory approach to data analysis.

Conclusion

Statistical literacy is an important skill for all students (Wild, 2017). As statistics is commonly taught within mathematics, statistical literacy needs to be developed in middle school mathematics classrooms. Here, textbooks play an important factor for the enacted curriculum. However, not much is known about the statistical content of mathematics textbooks in middle schools (Weiland, 2019).

To uncover the statistical literacy potential in mathematics textbooks, this study provided a specification of the learning content of statistical literacy through different activities concerning the selective and imaginative reading of statistical information (Büscher, in press). Selective reading refers to the process during which a phenomenon gets encoded into data, from which a model is abstracted, which is then interpreted in a statistical argument. During each of these steps, information gets lost. Imaginative reading refers to the process of intuiting this lost information in activities here denominated as de-interpreting, de-abstracting, and de-coding. Three German Grade 6 textbooks were then investigated in a qualitative document analysis study (Bowen, 2009). The results show that all textbooks overemphasize the selective reading activity of abstracting. Learning opportunities for other more conceptually demanding activities of statistical literacy do exist, but risk to be overshadowed by the possibly very technical activities of abstracting and de-abstracting.

These results can highlight the importance of the teacher for developing statistical literacy. The results show that learning opportunities can be heavily geared towards certain activities of statistical literacy, which is not sufficient for holistic statistical literacy. Teachers need to use textbooks as didactical instruments to provide and adapt them for their own goals. One the one hand, a skilled teacher could add a small question that elicits interpreting or de-interpreting to a task which itself only requires abstracting. The specification of the activities of statistical literacy might provide such a teacher with a conceptual framework for analysing and adapting their teaching. On the other hand, inexperienced teachers regarding statistics might very well overlook the potentials for developing statistical literacy, and studies have shown that teachers tend to adapt tasks mostly to make them "manageable", instead as to adapt the content of the tasks (van Steenbrugge & Ryve, 2018).

Limitations

The textbooks analysed here provide only a limited window into the development of statistical literacy in middle schools. Even in Germany, there are many regional differences in textbooks, and the selection of textbooks cannot be considered representative even for German schools. It also might well be the case that the potential for statistical literacy is different in the textbooks for grades other than Grade 6. Finally, as outlined above, there exist a difference between the textbook content and the curriculum enacted by teachers. This study does not claim empirical insights into the actual state of teaching statistical literacy in Germany or other countries. However, it does show that learning

opportunities for statistical literacy can vary wildly between different textbooks, and that textbooks run the danger of overemphasizing only small parts of statistical literacy. This study's specification of activities of selective and imaginative reading can provide a framework for researchers and teachers to evaluate learning opportunities for statistical literacy as well as an orientation for adapting their own teaching or didactic materials for fostering statistical literacy.

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