Conceptual learning opportunities in teachers' differentiated task designs for inclusive mathematics education

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Inclusive mathematics education creates new challenges to teachers, requiring additional knowledge and possibly changed classroom practices. One teaching job gaining importance is differentiating through task design, as teachers need to provide conceptually rich learning opportunities even to students with mathematical learning disabilities. However, more insight is needed into how teachers engage in this job. This study analyses teachers' designed tasks during a PD course to reconstruct their categories of differentiation for percentage problems. The observed teachers tend to differentiate in a way that excludes low-achieving students from conceptually rich learning opportunities on percentages. This can be explained by the differentiation strategy of splitting and partitioning teachers' envisioned ideal-typical solution paths to percentage problems.

Keywords: Teacher practices, inclusive education, percentages, differentiation, professional development.

Introduction

In recent years, a shift can be observed in research on teacher expertise from a focus on general dispositions or concrete performance towards a focus on situation-specific skills (Blömeke, Gustafsson, & Shavelson, 2015). This places an emphasis on the wide array of necessary tasks or jobs demanded of teachers in their classroom work, from small jobs such as choosing and illustrating examples (e.g. Fauskanger & Mosvold, 2017) to large jobs such as adapting textbooks for their own course (e.g. Priolet & Mounier, 2017).

Such a focus calls for careful attention given to the concrete situations of teachers' classroom work, as every situation imposes different situational demands. Recently, a new class of situations is gaining increased attention in mathematics education research: teaching situations involving inclusive mathematics education. In Germany and probably many other countries, large parts of mathematics teachers feel unprepared for dealing with these new situational demands. Research is needed into how teachers can face these situational demands of their teaching jobs in order to develop professional development (PD) courses to support teachers of inclusive mathematics classrooms.

This paper investigates the ways teachers engage with one central job of inclusive mathematics classrooms, namely the job of differentiating through task design in the context of percentage problems. First, this paper introduces a theoretical framing of the construct of teaching jobs, as well as a short overview on situational demands in inclusive mathematics education. The empirical part then illustrates how teachers use ideal-typical solution paths for differentiating through task design, involuntarily limiting conceptual learning opportunities for already low-achieving students.

Teaching jobs as a focus for research

For conceptualizing teachers' competences and practices, there exist a variety of conceptualizations and specification for these skills, such as 'core tasks and problems of teaching' (Bass & Ball, 2004) or 'core practices' (Forzani, 2014). Empirical research shows how each of these skills, such as choosing and illustrating examples, poses a complex interplay of situational demands to teachers (Fauskanger & Mosvold, 2017). What teachers need to know in order to master these tasks, however, so far remains underspecified.

Already over 25 years ago, Bromme (1992) identified this as a task for empirical research. Instead of providing a list of knowledge or dispositions, Bromme (1992) places specific situational demands (tasks of teaching) into the heart of teacher expertise. Each situational demand requires specific knowledge attained by teachers. This knowledge takes the form of categories, which "enable experts to discern a specific order within problematic situations or to construct it (e.g. by arranging instruction)" (ibid., p. 151, translated). It follows that PD programs need to support teachers in coping with their situational demands in teaching by supporting their development of categories.

This study follows the approach outlined by Prediger (submitted) to conceptualize teachers' situational demands as teaching *jobs* (in line with Bass & Ball, 2004) that are organized through teachers' personal *categories*. The empirical task remains to identify the jobs required of teachers by inclusive education and to empirically reconstruct their categories activated for mastering these jobs.

Conceptual learning in inclusive mathematics education

Since the ratification of the UN Convention on the Rights of Persons with Disabilities in 2008 (UN, 2006), inclusive education has seen an increased interest in educational systems across many countries and in mathematics education research. In Germany, ratification of the convention has resulted in a shift from an educational system in which students diagnosed with various disabilities were segregated from 'regular' students towards an inclusive system in which (most) students visit the same schools, with or without disabilities. This posed increased challenges to the majority of teachers who had little to none previous experiences with students with disabilities, creating a great demand for professional development (PD) of teachers for inclusive education.

Concerning mathematics education, the largest new group of students for teachers is the risk group of under-achieving students in mathematics, here referred to as students with MLD (mathematical learning disabilities/difficulties, for an overview on terminology see Scherer et al., 2016). MLD is a construct without a clear consensual definition, hindering making distinctions between biological, cognitive, and non-cognitive contributing factors (Lewis & Fisher, 2016). However, research in mathematics education indicates that students with MLD do not fundamentally differ in their learning from students without MLD: under constructivist perspectives on learning, both groups require conceptually rich learning situations in which they can draw on their own experiences in order to develop mathematical concepts (Scherer et al., 2016).

The teaching job of differentiating through task design

The increased heterogeneity of inclusive classrooms faces teachers with increased demands of differentiated instruction (Tomlinson & Moon, 2013). This heterogeneity can refer to a variety of aspects, including their learning resources (working memory, interest, language proficiency) and mathematical knowledge (informal experiences, conceptions, formal knowledge). Every learner needs instruction that recognizes his or her individual needs. Thus, one teaching job that is of increased importance for inclusive education is the job of *differentiating through task design*.

Differentiated task design can take several forms (see Tomlinson & Moon, 2013). This paper focuses on two forms in particular (for the third form of open-ended differentiation see Buró & Prediger, submitted): students with MLD might need conceptually rich tasks for content different from other students (i.e. differentiated in content). They also need tasks that pose demands that they are able to fulfill (i.e. differentiated in access). In turn, such differentiating tasks might also pose dangers, as they might alienate students with MLD by providing learning content different from that of the 'regular' students.

These categories for task design thus provide a first list of possible categories for teachers to draw upon in their job of differentiating through task design. However, reconstructing such categories remains an empirical task (Bromme, 1992), as teachers' personal categories might be richer than the mentioned ones. More insights are needed into how teachers enact this job of differentiating through task design, and how they ensure the conceptually rich learning opportunities required especially by students with MLD. This job here is treated as requiring topic-specific investigation, as differentiating in content requires topic-specific content knowledge. Because a majority of studies about MLD focus on elementary arithmetic (see Lewis & Fisher, 2016), this study instead opts to focus on a topic from secondary school, namely percentages. Thus, two research questions emerge for this paper:

(RQ1) What categories do teachers draw on when engaging in the job of differentiating through task design for percentage tasks?

(RQ2) How do these categories influence the learning opportunities for students with MLD in differentiated percentage tasks?

Methodology

Research was carried out during a PD course on inclusive education for the content of percentages, situated in the methodological approach of Design Research for teachers with a focus on content-specific professionalization processes (Prediger et al., 2016). Data were collected in this PD course, which is briefly introduced in this section.

Design of a PD course for inclusive mathematics education for percentages

The PD course aimed at in-service mathematics and special education teachers consisted of three distinct phases. In the first phase, the participants received information on didactic concepts and models as well as on concepts and models of special education. This phase consisted of three sessions (15 hours total) with the research team and participants. Topics included use of representations, fostering understanding for all, language-responsive teaching, working memory and learning, and automatization of routines. During this phase, the teachers were encouraged to discuss these concepts and to relate them to their own experiences. The second and third phases consisted of practice and reflection phases and are not part of this study.

Course-integrated instruments for eliciting teachers' categories

Investigation into teachers' categories was carried out through *course-integrated instruments*. These were instruments designed to collect written products by the teachers. However, they were not collected in a classical pre-post design, but rather were employed at various points throughout the PD sessions. These instruments served a dual role: on the one hand, they provided the data needed for research, so that they were used as *instruments* for professional development. On the other hand, they activated the participants' prior experiences and, after having been filled out, provided the basis for discussions, so that they could be used as *means* for professional development.

These instruments were designed to actualize the situational demands teachers face in inclusive classrooms in small situations, i.e. to elicit teaching jobs. Figure 1 shows the instrument used to elicit the job of differentiating through task design for percentages.

In the city, Maurice spotted a deal for the shoes of his dream. The deal is shown as a percentage bar.
All sneakers now only 75% of the old price!

Tasks

1. Please create three differentiating problems that refer to the situation depicted
2. For which students would you use which problem?

Figure 1: The course-integrated instrument for differentiating through task design for percentages (translated from German)

This instrument was given in the third session of the first phase of the PD course. It uses parts of a task on percentages already known to the teachers from the PD course. They also already were familiar with the basic ideas of the percentage bar, which was new to most of the teachers prior to the PD course (see Buró & Prediger, submitted).

Data collection and data analysis

The instrument was given to the teachers who had 15 minutes to create the differentiated tasks. Afterwards, a discussion took place and the written products were collected and digitized. Of the 17 products filled out by teachers, 1 was dropped for being too short for analysis. Data analysis then consisted in reconstructing three types of categories of differentiation: the types of differentiating tasks created by teachers, the perceived groups of students for which teachers create differentiation, and the mathematical learning opportunities provided to these groups through differentiation. The types of tasks could be deductively reconstructed as belonging either to tasks differentiated in content or in access. For the perceived groups and mathematical learning opportunities, categories were inductively generated by comparing and contrasting all teacher products. Categories were then refined and condensed, and then used in a deductive analysis in order to enable a comparison of the different written teacher products. As the categories thus also are an empirical result, they will be illustrated below.

Teachers' categories for differentiating through task design

Two teachers' products are printed in Figure 2. They can illustrate some common elements of the teachers' differentiations through task design which were reconstructed in the analysis. The first teacher, 'Anna' (left side), differentiates between four different levels (L-M-S-gS, interpreted as the initials of the German words for easy-medium-hard-very hard). This structuring is reconstructed as a differentiation into levels of 'low-middle-high' by combining the levels of 'S' and 'gS'. Her tasks, however, do not differ in complexity, but in content. Students who are assigned to the low level ('L') are tasked with simply reading the given values and with calculating a difference without any reference to percentages. Such instances are coded as *description* (reading, simple description of the involved numbers without explanation) and *subtraction*. Students in the middle level ('M') are assigned with a task that superficially does relate to percentages, but ultimately results again in a simple calculation of a difference. This again is a task of *subtraction*. Students in the high level ('S', 'gS') are tasked with filling out the missing steps in the percentage bar. This is a type of task that was discussed during the PD course as a task that can elicit *proportional* reasoning, often signified by 'simultaneously counting up' percentages and percent values until reaching the desired values. Finally, they are also tasked with a different *percentage* problem.

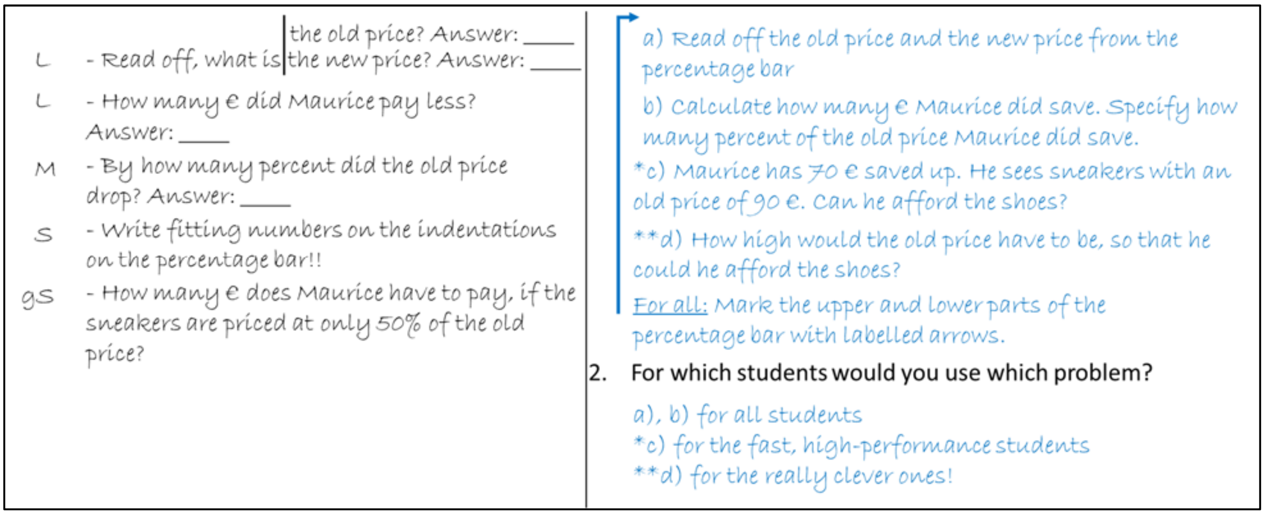

Figure 2: Two teacher differentiations through task design (translated from German)

The second teacher, 'Beatrice', chooses other categories for her differentiation. Whereas Anna does not make explicit if students are open to change levels as they see fit, Beatrice creates tasks in a progression in which students are expected to follow as far as they are able to, reconstructed as 'for all-middle-high'. She begins with a 'for all' category, which includes the task of *proportional* reasoning (counting up through labelled arrows) that Anna assigned to a much higher level. Her other two tasks a) and b) consist of *description* and *subtraction*. Students in the middle level are then given a different *percentages* problem, as are students in the high level (although a different, probably harder problem).

Comparing these two teachers reveals some interesting differences. In the differentiation of Anna, students with MLD would probably be assigned to the low level, and thus would receive learning opportunities for describing and subtraction, but not for percentages. Even if they would reach the next level, they still would be given learning opportunities for subtraction only. Such learning opportunities are also provided through the differentiation employed by Beatrice. However, through the category ‘for all’ she also includes a learning opportunity for proportional reasoning that students with MLD have access to – an important conceptual foundation for the learning content of percentages, which they could then utilize should they be able to reach the next level.

This worrying trend of lacking learning opportunity for percentages can also be observed throughout the differentiations of all teachers. Table 1 gives an overview on the learning content that teachers assign to their levels of differentiation.

	For all	**Low**	**Middle**	**High**
Description	6	10	3	1
Subtraction	1	3	3	1
Proportional	1	2	4	1
Explaining	0	1	4	6
Percentages	0	0	4	5

Table 1: Number of occurring learning contents for levels of differentiation (n=16)

The learning opportunities for students with MLD are reflected through the combination of the levels *for all* and *low* (the two grey columns). The learning contents that are conceptually relevant for percentages are represented through the categories *proportional*, *explaining*, i.e. the task of explaining the algorithm or procedure to find a percentage value, and *percentages* (the three grey rows). This shows a lack of conceptual learning opportunities for students with MLD (the dark grey intersection). Students with MLD are most commonly given tasks involving the rather passive description of the given situation or reading of values (16 occurrences). Of the remaining 8 occurrences, half consist of tasks involving simple subtraction. Only in 4 occurrences, these students have access to learning opportunities that involve conceptual understanding of percentages (proportional reasoning as foundation for percentages and explaining strategies for calculating percentage values).

The categories of Ideal-typical solution paths, partitioning, and conceptual foundations

For the aim of teacher PD, this bleak result of missing conceptual learning opportunities requires further explanation, i.e. an identification of the categories underlying the job of differentiating through task design. Anna can serve as an example representative of most teacher products. Her differentiation follows an observable structure: in order to fully understand the situation, students first need to read the relevant information (low level), then acknowledge and quantify the changing values (middle level), and finally use concepts of proportion and percentage to interpret the situation (high level). Breaking down this *ideal-typical solution path* results to her in five small subtasks. Her strategy now is to *partition* this ideal-typical solution path and to assign different tasks to different students. Unfortunately, this results in conceptually poor learning opportunities for students on the lower levels, as the conceptual heart of the percentage problem is placed at the end of the ideal-typical solution path, and thus only accessible for students assigned to the high level.

Beatrice follows a similar overall structure, but with different emphasis. Her ideal-typical solution path places proportional reasoning as the first part of the solution. This is an important *conceptual foundation* of percentage problems. Because her solution path is not neatly partitioned, but includes a level for all students, students with MLD also gain access to conceptual learning opportunities in this way.

Conclusions

Like all students, students with MLD require conceptually rich learning opportunities to develop understanding of key mathematical concepts such as percentages. For teachers new to this group of students, this increases the complexity of the teaching job of differentiating through task design. This study shows the ways teachers can enact this job, and how this can influence the learning opportunities students have access to. The empirical analysis shows how partitioning ideal-typical solution paths can result in students with MLD being denied access to conceptual learning opportunities on percentages, instead providing access to subtraction. Anna more useful approach is shown to provide access to conceptual foundations by including tasks that focus on proportional reasoning, to be approached by all students.

This study has followed the approach outlined by Bass & Ball (2004) to focus on teachers' tasks, namely the job of differentiating through task design. However, this study also illustrates that insights are needed into *how* teachers fulfill these tasks. The approach of identifying teachers' categories outlined by Bromme (1992) proved useful as a framework for describing how teachers fulfill their jobs.

So far, the categories of ideal-typical solution paths, partitioning, and conceptual foundations have been reconstructed from one course-integrated instrument during one PD course session. More insights are still needed to empirically ground and to refine these categories. Also, the missing conceptual learning opportunities need to be interpreted carefully: it might justifiably be the case that giving students with MLD access to learning subtraction is a necessary and reasonable decision, e.g. for students that still struggle with subtraction.

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